



The influence of the poloidal variation of the density on the locally measured velocities induced by biasing experiments

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Abstract

The plasma edge plays an important role in the physics of improved confinement. In this region, the shear of the radial electric field and the related rotation are thought to be responsible for the suppression of turbulence. We have developed a consistent set of experiments and theory to analyse these important phenomena. An electric field is set up with a biasing electrode. The resulting rotation velocities are measured with an inclined Mach probe. The measurements are then compared with the predictions of a theoretical fluid model. In this model, parallel viscosity and neutral friction were already identified as important components to explain the very important and localised electric fields. In this paper we focus on compressibility effects and show that it is necessary to take the poloidal variation of the density into account to explain the measured rotation velocities. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

It is well known that large poloidal and toroidal flows and very important, localised electric fields can be created in the edge plasma by electrode biasing. These flows, as they vary rapidly in space, are thought to be responsible for the suppression of turbulence and thus for the creation of transport barriers and improved confinement modes as the H-mode [1].

In an electrode biasing experiment, an electrode with its tip a few centimetres inside of the separatrix forces a radial current through the plasma. This current is the main driving force for the rotation. It is balanced by friction mechanisms for which we will retain as in [2,9] the parallel viscosity and interaction with neutrals. The importance of the convection term and of the perpen-

dicular viscosity has been treated in [2–5]. In this paper, we propose to assess the importance of the poloidal variation of the density (further referred to as ‘compressibility’) on the flow velocities as observed in TEXTOR-94.

We will establish a fluid model and confront the results predicted by this model with experimentally measured velocities and electric fields, both in L- and H-mode. These quantities are obtained with an inclined Mach probe as described in [6]. We will show that especially the toroidal velocity is influenced by compressibility. Because we can explain the measured toroidal rotation to a fair degree, we have an experimental verification of this effect, which was for the first time treated in [7,8]. The authors however omitted viscosity, resulting in shocks. Also Rozhansky and Tendler [9] estimated poloidal compressibility but using a kinetic approach. As their model does not contain the, in our opinion, important neutral friction force [2,10], it cannot be applied to the TEXTOR data.

In the next section we will briefly discuss our theory, while Section 3 is devoted to the comparison with the experiment. Conclusions are drawn in Section 4.

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2. Theory

We start with the fluid equations as given in [11]:

$$\vec{\nabla} \cdot (n\vec{V}) = 0, \quad (1)$$

$$\vec{\nabla} \cdot (m \cdot n \cdot \vec{V}\vec{V}) = -\vec{\nabla}p - \vec{\nabla} \cdot \vec{\pi} + (\vec{J} \times \vec{B}) + \vec{F}_{\text{neutrals}}, \quad (2)$$

where \vec{V} is the flow velocity, n the density, p the total pressure, $\vec{\pi}$ the viscosity tensor, \vec{J} the current density, m the ion mass, \vec{B} the magnetic field and the neutral drag force is given by $\vec{F}_{\text{neutrals}} = -v \cdot \vec{V}$, where v is a drag coefficient as defined in [12]. These equations will be used to compute the radial electric field (E_r) and the rotation velocities. We will work in a circular toroidal geometry (r, θ and ϕ are the radial, poloidal and toroidal co-ordinates) and neglect the Shafranov shift as we are working in the edge where it is small. Further, we assume that all quantities are composed of a poloidally constant part and a poloidally varying part, such that:

$$X(r, \theta) = \bar{X}(r) + \tilde{X}(r, \theta). \quad (3)$$

For the poloidally constant part we will take the flux surface average defined as

$$\bar{X}(r) = \langle X \rangle = \frac{1}{2\pi} \int \frac{R}{R_0} X(r, \theta) d\theta, \quad (4)$$

(R is the major radius) so that the variable part can be computed as $\tilde{X}(r, \theta) = X(r, \theta) - \bar{X}(r)$. We will assume that \tilde{X}/\bar{X} is at most of order $\varepsilon = r/R_0$. Corrections of $O(\varepsilon^2)$ will be neglected.

In a toroidal axi-symmetric geometry, the continuity equation reads

$$\frac{\partial}{\partial \theta} (R \cdot n \cdot V_\theta) = 0, \quad (5)$$

so that

$$V_\theta = \frac{F(r)}{R_0} \left(\frac{R_0}{R} - \frac{\tilde{n}}{\bar{n}} \right), \quad (6)$$

where $F(r)/R_0$ is an as yet unknown flux function expressing the average poloidal rotation.

The radial electric field will be supposed poloidally constant, because of the important parallel conductivity of the plasma. The radial ion momentum equation then results in

$$V_\perp = \frac{1}{B} \left(-E_r + \frac{1}{en} \frac{\partial p_i}{\partial r} \right) = \frac{1}{B} V(r) = \frac{R}{R_0} \cos \alpha \frac{V(r)}{B_0}, \quad (7)$$

where α is the angle between the parallel and toroidal directions and $V(r)/B_0$ is a second flux function introducing the electric field and related to the average perpendicular rotation. As the pressure gradient term in biasing experiments is always much smaller than the electric field, we will drop it for the moment. Note

however that this reduces the applicability of the formulas to the region where the field is sufficiently high.

Application of the projection relations then allows us to compute the toroidal velocity:

$$V_\phi = \frac{1}{\Theta} \left(\frac{F(r)}{R_0} \frac{R_0}{R} - \frac{V(r)}{B_0} \frac{R}{R_0} - \frac{\tilde{n}}{\bar{n}} \frac{F(r)}{R_0} \right) \quad (8)$$

with $\Theta = \sin \alpha / \cos \alpha$. We are now ready to exploit the momentum equation (Eq. (2)), the parallel projection of which, only retaining viscosity and neutral drag, reads

$$\left(\vec{B} \cdot \vec{\nabla} \cdot \vec{\pi} \right) + \left(v \vec{B} \cdot \vec{V} \right) = 0. \quad (9)$$

The surface averaged version of this equation gives a first relation between $F(r)$ and $V(r)$:

$$\left\langle \left(\vec{B} \cdot \vec{\nabla} \cdot \vec{\pi} \right) \right\rangle + \left\langle \left(v \vec{B} \cdot \vec{V} \right) \right\rangle = 0. \quad (10)$$

Multiplication with B is important as then the viscosity term can be written as

$$\left\langle \vec{B} \cdot \vec{\nabla} \cdot \vec{\pi} \right\rangle = B_0 \frac{3}{2} \frac{\Theta \bar{\eta}_0}{R_0^2} \left(\frac{F(r)}{R_0} - V_{\text{NEO}} \right), \quad (11)$$

with $V_{\text{NEO}} = -0.5(1/Z e \cdot B_0)(\partial T_i / \partial r)$ and

$$\eta_0 = m_i n R_0^2 \left(\frac{\sqrt{\pi}}{3} \frac{q \cdot V_{\text{thi}}}{R_0} e^{-U_{\text{pm}}^2} + \frac{1}{3} \frac{v_{\text{ii}} q^2}{1 + U_{\text{pm}}^2} \right) \quad (12)$$

and where $U_{\text{pm}} = -E_r / (B_0 V_{\text{thi}})$. The other symbols have their usual meaning.

Note that the neutral friction [12] and the viscosity coefficients (12) are proportional to the ion density; we will express this dependence explicitly by $v = nv^* = (\bar{n} + \tilde{n})v^*$ and $\eta_0 = m_i \eta_0^*$, while we also introduce the parameter $\Delta = \eta^* / v^*$.

The surface averaged toroidal projection of Eq. (2) introduces a second relation between the unknown functions $F(r)$ and $V(r)$:

$$I_r = S \langle J_r \rangle = \left\langle \frac{1}{B_\theta} n v^* V_\phi \right\rangle, \quad (13)$$

where S is the surface through which the current flows.

It is important to note that the contribution of the parallel viscosity vanishes from this expression [2,13].

The averaged equations (10) and (13) are to $O(\varepsilon^2)$ independent of \tilde{n} so that their solution yields $F(r)$ and $V(r)$ for an imposed radial current and known ion density, temperature and neutral density profiles.

In a second step we then solve Eq. (9) posing: $\tilde{n}/\bar{n} = \text{Real}(N_e e^{i\theta})$, resulting in:

$$\frac{\tilde{n}}{\bar{n}} = - \left\{ \left[1 + \frac{\Delta}{q^2 R_0^2} \right] \frac{F(r)}{R_0} \right\} / \left\{ \left[\frac{4}{3} \frac{\Delta}{q^2 R_0^2} \right] \frac{F(r)}{R_0} + \frac{V(r)}{B_0} \right\} 2\varepsilon \cos \theta. \quad (14)$$

3. Discussion

We propose to examine electrode biasing results obtained in TEXTOR-94. The discussed velocity profiles were obtained by an inclined Mach probe measurement as explained in [6]. Relevant parameters are $R_0 = 1.75$ m, $B_0 = 2.33$ T, $q(0) = 0.88$, $q(\text{Separatrix}) = 6.7$, $Te \cong 40$ eV, $Ti \cong 40$ eV, $n \cong 10^{19}$ m $^{-3}$. The biasing electrode penetrates 5 cm inside of the separatrix, with a conducting tip of 1.5 cm. We will examine an L-mode for which the electrode current is $I_r = 163$ A, and an H-mode for which we have $I_r = 110$ A.

We can now compare the experimental results with our model. For the temperature and ion density profiles we take experimental values while the neutral density is modelled by an exponentially declining function towards the centre: $n_n = n_{no} \exp(-(r-a)/\lambda)$, where n_{no} is the neutral density at the separatrix, λ the decay length and a is the radial position of the separatrix. The parameters n_{no} and λ are adjusted so as to have a good agreement between the measured and computed electric field. We then compute the velocities (V_θ and V_ϕ) and compare them with the experimental results.

A first important observation is that the electric field peaks sharply in the vicinity of the separatrix (Fig. 1) and becomes small again well inside of the region where the radial current is constant (-3.5 cm $< r - a < 0$ cm). This behaviour can qualitatively be understood when $V(r)$ is eliminated between Eqs. (10) and (13), resulting in:

$$\begin{aligned} -\bar{n} \frac{F(r)}{R_0} \left[\eta^* \frac{3}{2R_0^2} + v^*(1 + 2q^2) \right] \\ = I_r \frac{B_0}{S} - V_{\text{NEO}} \frac{3 \bar{n} \eta^*}{2 R_0^2}. \end{aligned} \quad (15)$$

Note that $F(r)/R_0$ (the average poloidal rotation) is almost equal to $V(r)/B_0$ (the average perpendicular rotation) because the angle between the poloidal and the perpendicular directions is small. Therefore, the following discussion is valid for both quantities. Note also that $-V(r)$ is related to the radial electric field by means of Eq. (7). At the right-hand side of Eq. (15) we recognise the driving terms and note that the second one ($-V_{\text{NEO}}$) is much smaller than the first one in the case of biasing. Furthermore, I_r is constant in the region where the field peaks, so that the left-hand side of the equation must also be constant.

At the left-hand side we recognise the factor $\eta^*(3/2R_0^2) + v^*(1 + 2q^2)$, the first term of which represents the viscosity η^* which decreases slightly towards the separatrix because of the decreasing temperature. The second term contains the factor $(1 + 2q^2)$ which increases a few percent in the interval of about 1 cm where the electric field is high. The second factor, representing the neutral friction, also increases because it is proportional to the neutral density. Fig. 2 however depicts the ion and neutral densities used in the H-mode. From this figure it is clear that the ion density decreases much more rapidly than the neutral density increases. Therefore, $F(r)$ must increase. But an increase in $F(r)$ and thus in E_r , means a reduction of the viscosity (see Eq. (12)) so that, according to Eq. (15), $F(r)$ must further increase until finally kept in check by neutral friction. The same arguments are valid for the L-mode, though here all the effects are less extreme. It is now also clear that n_{no} is directly related to the value of E_r at the separatrix. In H-mode, as the electric field is higher than in L-mode, n_{no} must be smaller ($n_{no,L} = 1.5 \times 10^{17}$ m $^{-3}$, $n_{no,H} = 0.3 \times 10^{17}$ m $^{-3}$). This result was already confirmed in [10].

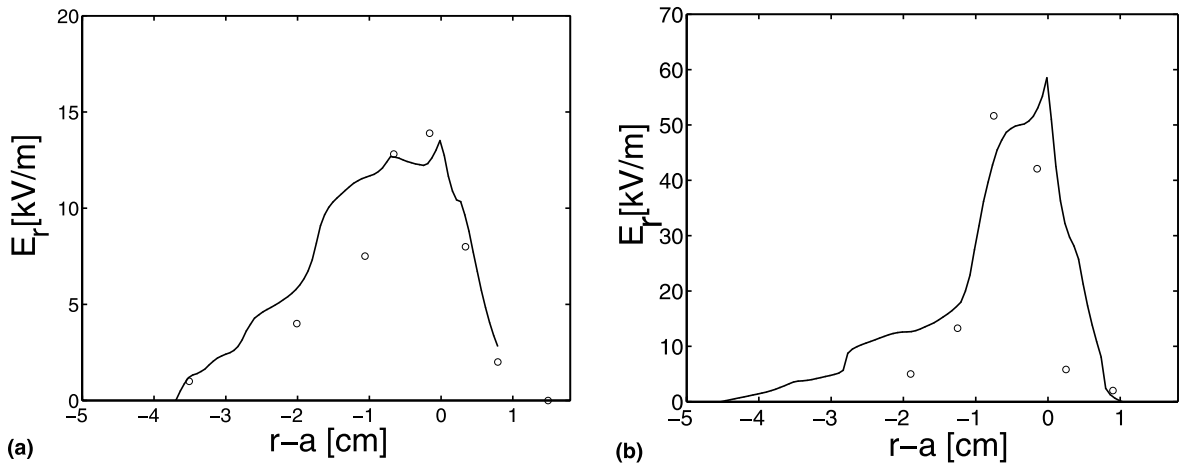


Fig. 1. (a) Electric field E_r vs. minor radius $r - a$ in L- and H-mode. (b) Comparison between measured (dots) and calculated E_r (solid line).

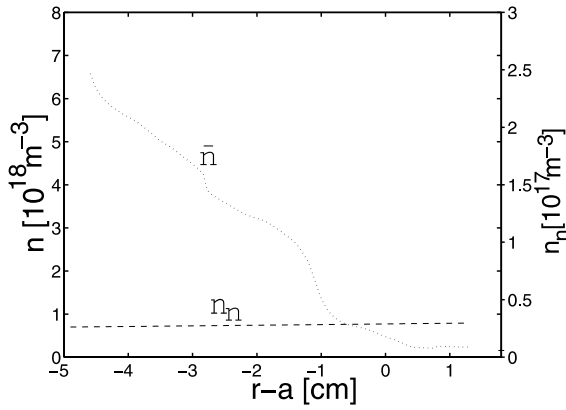


Fig. 2. Ion and neutral density in the H-mode.

The sharp increase in E_r will quench turbulence because of the shear in the related velocities, and thus reduce radial transport so that the density further drops at

the location of the slope of the field. This effect, according to Eq. (15), again helps to steepen the electric field.

Once inside of the scrape off layer, I_r is no longer constant as it is deviated towards the limiter. The shape of the electric field and of the rotation velocities is then largely set by the decreasing radial current.

We now turn our attention to the toroidal rotation, the main feature of which is that it peaks at a different radial location than E_r , as shown in Figs. 3(a) and (b), depicting the measured toroidal velocities at the out-board equatorial plane ($\theta = 0$). In V_ϕ , the influence of compressibility is most strongly noted. Indeed, when $\tilde{n}/n = 0$, Eq. (8) reduces to (we take $F(r)/R_o \cong V(r)/B_o$ for this argument):

$$V_\phi = \frac{1}{\Theta} \left[\frac{F(r)}{R_o} \frac{R_o}{R} - \frac{V(r)}{B_o} \frac{R}{R_o} \right] \approx -2q \frac{V(r)}{B_o} \cos \theta, \quad (16)$$

which means that V_ϕ would be very large and would have the same shape as the electric field (see Figs. 3(a)

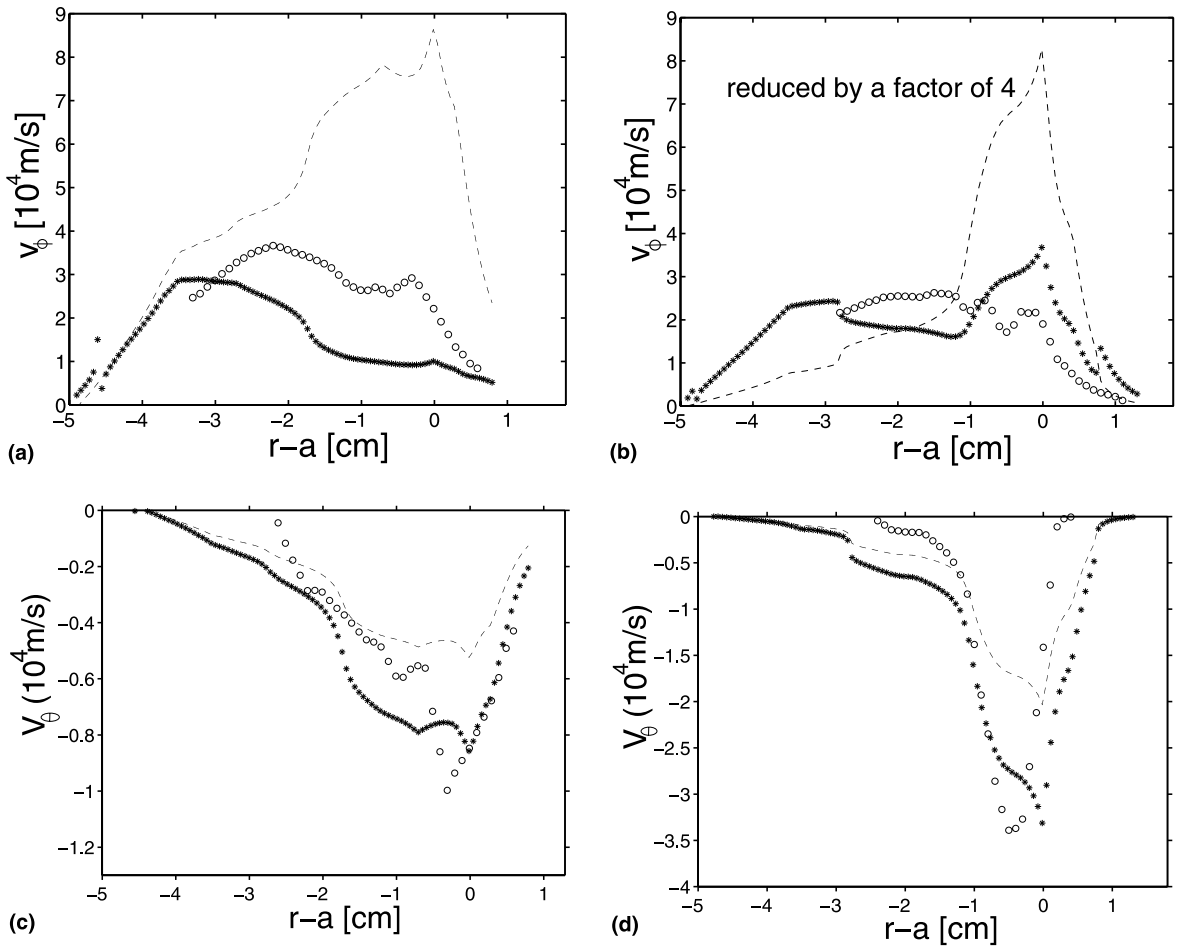


Fig. 3. (a–d) v_ϕ and v_θ vs. minor radius $r - a$ in L- and H-mode. Comparison between measured (\circ) and calculated velocities, with \tilde{n}/\bar{n} (*) and without \tilde{n}/\bar{n} (dashed line).

and (b), upper curves, and note that the calculated velocity is reduced by a factor 4 in the case of the H-mode). When now compressibility is added, we see that Eq. (14) reduces to

$$\frac{\bar{n}}{\bar{n}} = -2\varepsilon \cos \theta, \quad (17)$$

when the electric field is high and Δ goes to zero. Eq. (8) then becomes

$$\begin{aligned} V_\phi &\approx \frac{1}{\Theta} \left[\frac{F(r)}{R_o} (1 - \varepsilon \cos \theta) - \frac{V(r)}{B_o} \right. \\ &\quad \left. \times (1 + \varepsilon \cos \theta) + 2\varepsilon \cos \theta \frac{F(r)}{R_o} \right], \\ &\approx (1 + \varepsilon \cos \theta) \langle V_\phi \rangle. \end{aligned} \quad (18)$$

The result is that the local velocity behaves in the same way as the average toroidal rotation, while the strong effect of E_r is eliminated. The thus obtained velocities are also depicted in Figs. 3(a) and (b) and show much better agreement with the experiment, in the H-mode as well as in the L-mode.

According to Eq. (13), $\langle V_\phi \rangle$ should furthermore vanish when I_r drops to zero at the tip of the electrode which is also reproduced in the plots.

Figs. 3(c) and (d) show the effect of compressibility on the (local) poloidal velocity. The effect here is less pronounced than on the toroidal velocity. Nevertheless it is clear that the inclusion of compressibility gives better agreement with the experiment.

Finally we are able to compute the poloidal variation of the radial current density from Eq. (2), the toroidal projection of which, together with Eqs. (8) and (17), reduces to

$$J_r = \frac{\bar{n}v^*}{B_o\Theta} \langle V_\phi \rangle (1 - 3\varepsilon^2 \cos^2 \theta), \quad (19)$$

showing that J_r is poloidally constant to $O(\varepsilon^2)$.

4. Conclusions

We can conclude that our model, which includes neutral friction parallel viscosity reduced by the electric field and compressibility, is capable of explaining the measured velocities as well as the electric field, not only qualitatively, but also quantitatively. The effect of compressibility is most noteworthy in the toroidal velocity. Although its potential significance in the case of strong rotation was already mentioned in [9], we have shown here a clear experimental proof.

As in our theory the neutral density is the only fitting parameter, the measurement of the electric field or of the toroidal rotation can be used to deduce the neutral density.

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